Nonforward anomalous dimensions of Wilson operators in $N = 4$ super-Yang-Mills theory.

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Abstract

We present the next-to-leading order results for universal non-forward anomalous dimensions of Wilson twist-2 operators in N=4 supersymmetric Yang-Mills theory. The whole calculation was performed using supersymmetric Ward identities derived in this paper together with already known QCD results and does not involve any additional calculation of diagrams. We also considered one particular limit of our result, which could potentially be interesting in the context of AdS/CFT correspondence.

1 Introduction

Parton distributions in QCD satisfy the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [1] and Dokshitzer-Gribov-Lipatov-Altarelli-Parizi (DGLAP) [2] equations. Up to day even next-to-leading corrections to these equations are known [3, 4]. Moreover, recently, similar results were also obtained for supersymmetric theories [5, 6, 7]. An idea to consider these equations in the case of supersymmetric theories is based on common expectations, that the presence of extra high symmetry may significantly simplify them as well as their analysis. For example, it was already known, that all quasi-partonic operators in $N = 1$ SYM form supermultiplet of operators [8], having the same single universal anomalous dimension with shifted argument, which at that time was computed in leading order (LO) of perturbation theory. Calculations in $N = 4$ SYM gave even more remarkable results - the eigenvalues of the integral kernels in the evolution equations for quasi-partonic operators are proportional to $\Psi(j-1) - \Psi(1)$ [9, 10], which means that these evolution equations in the multicolour limit are equivalent to the Schrödinger equation for the integrable Heisenberg spin model [9] similar to the one found in the Regge limit [11]. Moreover, it was shown, that in the maximally supersymmetric $N = 4$ Yang-Mills theory there is a deep relation between BFKL and DGLAP evolution equations [6]. In particular, the anomalous dimensions of Wilson twist-2 operators in $N = 4$ SYM could be found from the eigenvalues of the kernel of BFKL equation. The corresponding next-to-leading order (NLO) calculations showed, that many of these findings are valid also in higher orders of perturbation theory. Moreover, as in leading order they most fully realize in maximally supersymmetric $N = 4$ SYM.

However, some of the properties of these equations, valid at leading order, are violated at higher orders. For example, the conformal invariance of the theory in leading order allows us to construct multiplicatively renormalized quasi-partonic operators [8] up to this order in perturbation theory. But, in next-to-leading order in perturbation theory multiplicative renormalization of conformal operators is violated due to necessity in regularization of arising ultraviolet divergences, which is responsible for violation of conformal symmetry.

The Efremov-Radyushkin-Brodsky-Lepage (ER-BL) equation [12] may be viewed as some kind of the generalization of the DGLAP equation for the case of non-forward distribution functions. In this case in and out hadronic states in matrix elements of operators are different, what allows us to study scale properties of hadron wave functions. In the latter case the matrix elements of corresponding operators are considered between vacuum and hadronic state. In leading order the kernel of ER-BL evolution equation, after Mellin transformation was performed, is diagonal in basis of Gegenbauer polynomials [13, 14]. This is equivalent to the fact, that multiplicatively renormalized operators of ER-BL evolution kernel coincide in leading order with conformal operators constructed from Gegenbauer polynomials [15]. The direct calculation of next-to-leading order corrections to the evolution kernel of this equation turned out to be much more difficult problem compared to the forward case. It was first performed for the case of non-singlet quark operator in [16, 13], where in the last reference an advanced computation method was developed and some analysis of general properties of evolution kernel and corresponding anomalous dimension matrix was given. In this case as well as in the case of $\phi^3_{[6]}$ model a solution of ER-BL evolution equation was constructed at next-to-leading order of perturbation theory [17]. It was noticed, that at next-to-leading order conformal invariance is violated and anomalous dimension matrix develops non-diagonal part in the basis of conformal operators.

The source of conformal symmetry breaking was identified later in [18, 19]. It was found, that non-diagonal part of anomalous dimension matrix arises entirely due to the violation of special conformal symmetry. Moreover, a framework based on the analysis of broken conformal Ward identities was proposed, which allowed to obtain for the first time next-to-leading order corrections to the nonforward anomalous dimension matrix in singlet case both in QED [20] and in QCD [21]. The generalization of these results for the case of supersymmetric theories involves a number of relations between elements of anomalous dimensions matrix [22], some of which were already known from the analysis of forward limit (last paper in Ref. [2],[8]). Later, the use of these supersymmetry relations, which could be also easily written for the corresponding evolution kernel matrix [8, 22], allowed to determine for the first time the QCD ER-BL evolution kernels in the singlet case both for odd [23] and even [24] parity distribution amplitudes (see also [25]).

In this paper we, using already known results from QCD, found closed analytical expression for universal non-forward anomalous dimension for maximally supersymmetric $N = 4$ Yang-Mills theory. Earlier, the authors of [6] used already known QCD results [4] to obtain expression for NLO universal forward anomalous dimension of Wilson twist-2 operators in $N = 4$ SYM, which were later confirmed by direct calculations in Ref. [7]. A problem, we are solving here, may be formulated as follows: how knowing QCD results one may obtain analogous results in supersymmetric theory with minimum efforts.

It turns out, that owing to remarkable properties of $N = 4$ SYM it is possible to derive an expression for universal non-forward anomalous dimension without any additional calculation beyond those already done for QCD.

Our result could be also interesting in the context of AdS/CFT correspondence [26]. Namely,

there are some calculations of the anomalous dimensions of such operators in the limit of large j (Lorentz spin) from both sides of the AdS/CFT correspondence [27, 28, 29, 30, 7, 31]. There are, also some predictions for anomalous dimensions of other types of operators corresponding to multi-spin solutions in $AdS_5 \times S_5$ space from string theory side [32] [1], partially confirmed by field theory calculations.

It should be noted, that up to this moment only diagonal part of anomalous dimension matrix have been studied in the context of AdS/CFT correspondence and it would be interesting to compare the results for its non-diagonal part with the appropriate result from string theory.

The present paper is organized as follows. First in next two subsections we define the lagrangian of $N = 4$ SYM and introduce singlet conformal operators present in this theory. Next, in section 2 we derive supersymmetric Ward identities and resulting constrains on anomalous dimensions of our conformal operators. Then, in section 3 we proceed with the determination of universal non-forward anomalous dimension for Wilson twist-2 operators in $N = 4$ SYM. And, finally, section 4 contains our conclusion.

1.1 Lagrangian of $N = 4$ SUSY YM

Different supersymmetric Yang-Mills theories in four dimension can be constructed from higher dimensional $N = 1$ supersymmetric Yang-Mills theory through dimensional reduction [33]. To obtain the lagrangian of $N = 4$ SYM in 4-dimensions we start with $N = 1$ SYM in 10-dimensions

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \frac{i}{2} \bar{\lambda}^a \Gamma_\mu D^\mu \lambda^a. \tag{1}$$

Here Γ_μ are 10-dimensional Dirac gamma-matrices, $G^a_{\mu\nu}$ is gauge field strength, D^μ is covariant derivative and λ^a is Majorana-Weyl spinor. The supersymmetry transformations are

$$\begin{aligned} \delta^Q \mathcal{A}^a_\mu &= i\,\bar{\xi}\Gamma_\mu \lambda^a, \\ \delta^Q \lambda^a &= \Sigma_{\mu\nu} G^{a\,\mu\nu} \xi\,, \end{aligned} \tag{2}$$

where $\Sigma_{\mu\nu} = \frac{1}{4}[\Gamma_\mu, \Gamma_\nu]$. After dimensional reduction to four dimensions we get the following lagrangian of $N = 4$ supersymmetric Yang-Mills theory [34]

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \frac{i}{2} \bar{\lambda}^a \gamma_\mu D^\mu \lambda^a + \frac{1}{2}\left(D_\mu A^a_r\right)^2 + \frac{1}{2}\left(D_\mu B^a_r\right)^2 \\ &- \frac{g}{2} f^{abc} \bar{\lambda}^a \left(\alpha_r A^b_r + \gamma_5 \beta_r B^b_r\right) \lambda^c \\ &- \frac{g^2}{4}\left[\left(f^{abc} A^b_r A^c_t\right)^2 + \left(f^{abc} B^b_r B^c_t\right)^2 + 2\left(f^{abc} A^b_r B^c_t\right)^2\right], \end{aligned} \tag{3}$$

where λ^a denotes vector of 4 Majorana spinors and

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g f^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu, \\ D_\mu \lambda^a &= \partial_\mu \lambda^a + g f^{abc} \mathcal{A}^b_\mu \lambda^c, \end{aligned}$$

$$\alpha_1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}, \qquad \alpha_2 = \begin{pmatrix} 0 & -\sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, \qquad \alpha_3 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix},$$

[1] see last reference in [32] for review

$$\beta_1 = \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix}, \qquad \beta_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \beta_3 = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix},$$

$$\sigma_{\mu\nu} = \frac{1}{4}\left[\gamma_\mu, \gamma_\nu\right], \qquad \sigma'_{rt} = \frac{1}{4}\left[\alpha_r, \alpha_t\right], \qquad \sigma_{rt} = \frac{1}{4}\left[\beta_r, \beta_t\right], \qquad \kappa_{rt} = \frac{1}{4}\left\{\alpha_r, \beta_t\right\}.$$

In 4-dimensions we have gauge field (gluon), four Majorana fermions (gauginos), three scalars (A) and three pseudoscalars (B). Our notation is somewhat different form usually employed $SU(4)$-covariant form of $N = 4$ SYM, where we have gauge field, four left Weyl fermions and 6 real scalars. Certainly, both these forms of $N = 4$ lagrangian are completely equivalent and we have chosen this one only because it seems to be more convenient in doing actual loop calculations. The 4-dimensional supersymmetry transformation (2) now become (here ξ stands for a vector of 4 Majorana spinors)

$$\begin{aligned}
\delta \mathcal{A}^a_\mu &= i\bar{\xi}\gamma_\mu\lambda^a, \\
\delta A^a_r &= \bar{\xi}\alpha_r\lambda^a, \\
\delta B^a_r &= \bar{\xi}\gamma_5\beta_r\lambda^a, \\
\delta\lambda^a &= \left[\sigma_{\mu\nu}G^{a\,\mu\nu} + i\gamma_\mu D^\mu\left(\alpha_r A^a_r + \gamma_5\beta_r B^a_r\right)\right. \\
&\qquad \left. + g f^{abc}\left(\sigma'_{rt}A^b_r A^c_t + \sigma_{rt}B^b_r B^c_t + 2\gamma_5\kappa_{rt}A^b_r B^c_t\right)\right]\xi, \\
\delta\bar{\lambda}^a &= \bar{\xi}\left[-\sigma_{\mu\nu}G^{a\,\mu\nu} + i\gamma_\mu D^\mu\left(\alpha_r A^a_r - \gamma_5\beta_r B^a_r\right)\right. \\
&\qquad \left. - g f^{abc}\left(\sigma'_{rt}A^b_r A^c_t + \sigma_{rt}B^b_r B^c_t - 2\gamma_5\kappa_{rt}A^b_r B^c_t\right)\right].
\end{aligned} \tag{4}$$

Note, if we choose $\lambda_3 = \lambda_4 = 0$, λ_1, λ_2 non-vanishing and $A_1 = A_2 = B_1 = B_2 = 0$, $A_3 = A$, $B_3 = B$ non-vanishing, then using the explicit representation of α and β matrices lagrangian (3) becomes lagrangian for $N = 2$ SUSY theory.

Moreover, $N = 4$ SYM has internal $SU(4)$ R-symmetry group, which is symmetry of lagrangian under the following transformations of fields

$$\begin{aligned}
\delta \mathcal{A}^a_\mu &= 0, \\
\delta A^a_r &= \Lambda'_{rt}A^a_t + \widetilde{\Lambda}_{rt}B^a_t, \\
\delta B^a_t &= \Lambda_{rt}B^a_r - \widetilde{\Lambda}_{rt}A^a_r, \\
\delta\lambda^a &= -\frac{1}{2}(\sigma'_{rt}\Lambda'_{rt} + \sigma_{rt}\Lambda_{rt} + 2\gamma_5 k_{rt}\widetilde{\Lambda}_{rt})\lambda^a, \\
\delta\bar{\lambda}^a &= \frac{1}{2}\bar{\lambda}^a(\sigma'_{rt}\Lambda'_{rt} + \sigma_{rt}\Lambda_{rt} - 2\gamma_5 k_{rt}\widetilde{\Lambda}_{rt}),
\end{aligned} \tag{5}$$

where parameters of these transformations, given by real antisymmetric matrices Λ'_{rt}, Λ_{rt} and $\widetilde{\Lambda}_{rt}$.

Later in this paper we will be interested in scale transformation properties of Wilson operators of lowest twist or, what is the same, in those, which have maximal Lorentz spin. The component with maximal Lorentz spin has a symmetric traceless Lorentz structure and the simplest way to project it out is to use a convolution with the product of light-like vectors n_μ ($n^2 = 0$). This way we effectively project our theory on the light-cone. For a general four-vector X^μ we introduce light-cone coordinates as follows: $X^\pm = 1/\sqrt{2}(X^0 \pm X^3)$ and $X^\mu Y_\mu = X^+Y^- + X^-Y^+ - X^iY^i$, where $i = 1, 2$. For spinors the appropriate projectors are $\lambda^\pm = \frac{1}{2}\gamma^\pm\gamma^\mp\lambda$, so that $\lambda = \lambda^+ + \lambda^-$. At this step it is also convenient to fix gauge condition. We employ light-cone gauge defined by setting $\mathcal{A}^{a\,+} = 0$. Then taking $\xi = \xi^+$ (so that $\delta\mathcal{A}^+ = 0$) the restricted light-cone supersymmetry transformations

(4) become ($\gamma_\mu^\perp = \gamma_\mu - n_\mu \gamma_- - n^*_\mu \gamma_+$, where n_μ and n^*_μ project "plus" and "minus" components respectively) [35]

$$
\begin{aligned}
\delta^Q \mathcal{A}_\mu^{a\,\perp} &= i\bar{\xi}^+ \gamma_\mu^\perp \lambda^{a\,-}, \qquad \delta^Q A^a = \bar{\xi}^+ \lambda^{a\,-}, \qquad \delta^Q B^a = \bar{\xi}^+ \gamma_5 \lambda^{a\,-}, \\
\delta^Q \lambda^{a\,-} &= -\gamma^- \gamma_\mu^\perp \partial^+ \mathcal{A}_\mu^{a\,\perp} \xi^+ + i\gamma^- \partial^+ (A^a + \gamma_5 B^a)\xi^+, \\
\delta^Q \bar{\lambda}^{a\,-} &= \bar{\xi}^+ \gamma^- \gamma_\mu^\perp \partial^+ \mathcal{A}_\mu^{a\,\perp} + i\bar{\xi}^+ \gamma^- \partial^+ (A^a - \gamma_5 B^a).
\end{aligned}
\tag{6}
$$

In light-cone gauge the restricted supersymmetry transformations form the off-shell supersymmetry algebra. They are linear and form a closed algebra on the projected +-components of fields, defined as components having maximal spin.

1.2 Conformal twist-2 operators in $N = 4$ SUSY YM

Now, let us introduce the local singlet (with respect to internal SU(4)-symmetry group) conformal Wilson twist-2 operators appearing in this models for unpolarized and polarized cases [15, 8, 22, 36]

$$
\mathcal{O}^G_{j,l} = G^{a\perp}_{+\mu} (i\partial_+)^{l-1} C^{5/2}_{j-1} \left(\frac{\mathcal{D}_+}{\partial_+} \right) g^\perp_{\mu\nu} G^{a\perp}_{\nu +}\,, \tag{7}
$$

$$
\widetilde{\mathcal{O}}^G_{j,l} = G^{a\perp}_{+\mu} (i\partial_+)^{l-1} C^{5/2}_{j-1} \left(\frac{\mathcal{D}_+}{\partial_+} \right) \epsilon^\perp_{\mu\nu} G^{a\perp}_{\nu +}\,, \tag{8}
$$

$$
\mathcal{O}^\lambda_{j,l} = \frac{1}{2} \bar{\lambda}^a_{+\,i} (i\partial_+)^{l} \gamma_+ C^{3/2}_{j} \left(\frac{\mathcal{D}_+}{\partial_+} \right) \lambda^{a\,i}_+\,, \tag{9}
$$

$$
\widetilde{\mathcal{O}}^\lambda_{j,l} = \frac{1}{2} \bar{\lambda}^a_{+\,i} (i\partial_+)^{l} \gamma_+ \gamma_5 C^{3/2}_{j} \left(\frac{\mathcal{D}_+}{\partial_+} \right) \lambda^{a\,i}_+\,, \tag{10}
$$

$$
\mathcal{O}^\phi_{j,l} = \bar{\phi}^a_r (i\partial_+)^{l+1} C^{1/2}_{j+1} \left(\frac{\mathcal{D}_+}{\partial_+} \right) \phi^a_r\,, \tag{11}
$$

where $\phi = A + iB$ is a complex scalar field, $\mathcal{D} = \overrightarrow{\partial} - \overleftarrow{\partial}$, $\partial = \overrightarrow{\partial} + \overleftarrow{\partial}$, $g^\perp_{\mu\nu} = g_{\mu\nu} - n_\mu n^*_\nu - n_\nu n^*_\mu$, $\epsilon^\perp_{\mu\nu} \equiv \epsilon^{\alpha\beta\rho\sigma} g^\perp_{\alpha\mu} g^\perp_{\beta\nu} n^*_\rho n_\sigma$ and $C^\nu_n(z)$ are Gegenbauer polynomials

$$
C^\nu_n(z) = \frac{(-1)^n 2^n}{n!} \frac{\Gamma(n+\nu)}{\Gamma(\nu)} \frac{\Gamma(n+2\nu)}{\Gamma(2n+2\nu)} (1-z^2)^{-\nu+1/2} \frac{d^n}{dz^n} \left[(1-z^2)^{n+\nu-1/2} \right]. \tag{12}
$$

In what follows we will restrict ourselves to the analysis of singlet conformal operators, just introduced, and tensor operator to be introduced in next section. However, in general, $N = 4$ SYM has much richer content of twist-2 conformal operators, sitting in different representations of $SU(4)$-group. Besides bosonic operators in other irreducible representations of $SU(4)$-group, we can write down fermionic (by quantum numbers) operators formed by scalar-gluino and gluon-gluino fields. While these fermionic operators were already present in theories with less supersymmetry like $N = 1$ SYM [8, 22] and Wess-Zumino model [36], here we encounter new type of operator - vector operator formed by scalar and gluon fields. Under restricted supersymmetry transformations the conformal twist-2 operators form a closed operator supermultiplet. A general procedure of constructing supermultiplets of conformal operators is known for a long time [8]. Recently, full set of twist-2 conformal operators together with their transformations under restricted susy transformations was derived for the case of $N = 4$ SYM in Ref. [37] and we refer interested reader to that paper.

As was already mentioned in introduction, we will be interested in the renormalization properties of these operators. It should be noted, that in the singlet case there is mixing between bosonic operators formed by gluon, gluino and scalar fields (7)-(11). Also, in the non-forward kinematics, in contrast to forward case, the operators (7)-(11) will mix under renormalization not only with each other, but also with the total derivatives of themselves.

2 Supersymmetric Ward Identity in $N = 4$ SUSY YM

To begin with, let us introduce the following multiplicatively renormalized combinations of conformal operators for unpolarized (Eqs. (7), (9) and (11)) and polarized (Eqs. (8) and (10)) cases

$$\mathcal{S}^1_{j,l} = 6\mathcal{O}^g_{j,l} + \frac{j}{2}\mathcal{O}^\lambda_{j,l} + \frac{j(j+1)}{4}\mathcal{O}^\phi_{j,l}\,, \tag{13}$$

$$\mathcal{P}^1_{j,l} = 6\widetilde{\mathcal{O}}^g_{j,l} + \frac{j}{2}\widetilde{\mathcal{O}}^\lambda_{j,l}\,, \tag{14}$$

$$\mathcal{S}^2_{j,l} = 6\mathcal{O}^g_{j,l} - \frac{1}{4}\mathcal{O}^\lambda_{j,l} - \frac{(j+1)(j+2)}{12}\mathcal{O}^\phi_{j,l}\,, \tag{15}$$

$$\mathcal{P}^2_{j,l} = 6\widetilde{\mathcal{O}}^g_{j,l} - \frac{j+3}{2}\widetilde{\mathcal{O}}^\lambda_{j,l}\,, \tag{16}$$

$$\mathcal{S}^3_{j,l} = 6\mathcal{O}^g_{j,l} - \frac{j+3}{2}\mathcal{O}^\lambda_{j,l} + \frac{(j+2)(j+3)}{4}\mathcal{O}^\phi_{j,l}\,, \tag{17}$$

where the coefficients in front of operators can be found in a way, similar to [38, 8]. Here, we would like to note, that these linear combinations of conformal operators, which are also the components of $N = 4$ operator supermultiplet, renormalize multiplicatively only in LO of perturbation theory. Beyond LO in dimensional reduction, which preserves supersymmetry to rather high order in perturbation theory, used in this paper they get additional rotations due to the breakdown of superconformal symmetry. So, in general, it is only the constrains on anomalous dimensions of our conformal operators following from supersymmetric Ward identities, that remain valid to all orders of perturbation theory.

To derive the supersymmetric Ward identities, relating anomalous dimensions of singlet conformal operators to the anomalous dimension of some supersymmetry primary operator, which renormalized multiplicatively, we need to know the action of four restricted supersymmetry transformations on these singlet operators. Applying four restricted supersymmetry transformations to our initial singlet operators we get [2]: ($\delta^Q = \delta^Q_4\delta^Q_3\delta^Q_2\delta^Q_1$)

$$\delta^Q\mathcal{S}^1_{j,l} = \frac{1}{2}(1-(-1)^j)\bar{\xi}_4\gamma^\perp_\nu\gamma^-\xi_3\bar{\xi}_2\gamma^\perp_\mu\gamma^-\xi_1\ \mathcal{W}^{\mu\nu}_{j-2,l}\,, \tag{18}$$

$$\delta^Q\mathcal{S}^2_{j,l} = \frac{1}{2}(1-(-1)^j)\bar{\xi}_4\gamma^\perp_\nu\gamma^-\xi_3\bar{\xi}_2\gamma^\perp_\mu\gamma^-\xi_1\ \mathcal{W}^{\mu\nu}_{j,l}\,, \tag{19}$$

$$\delta^Q\mathcal{S}^3_{j,l} = \frac{1}{2}(1-(-1)^j)\bar{\xi}_4\gamma^\perp_\nu\gamma^-\xi_3\bar{\xi}_2\gamma^\perp_\mu\gamma^-\xi_1\ \mathcal{W}^{\mu\nu}_{j+2,l}\,, \tag{20}$$

$$\delta^Q\mathcal{P}^1_{j,l} = \frac{1}{2}(1+(-1)^j)\bar{\xi}_4\gamma^\perp_\nu\gamma^-\xi_3\bar{\xi}_2\gamma^\perp_\mu\gamma^-\xi_1\ \widetilde{\mathcal{W}}^{\mu\nu}_{j-1,l}\,, \tag{21}$$

$$\delta^Q\mathcal{P}^2_{j,l} = \frac{1}{2}(1+(-1)^j)\bar{\xi}_4\gamma^\perp_\nu\gamma^-\xi_3\bar{\xi}_2\gamma^\perp_\mu\gamma^-\xi_1\ \widetilde{\mathcal{W}}^{\mu\nu}_{j+1,l}\,, \tag{22}$$

[2]Supersymmetric transformations for all operators in $N = 4$ SYM can be found in Appendix of Ref.[37]

where

$$\mathcal{W}^{\mu\nu}_{j,l} = \tau^{\mu\nu,\rho\sigma}_{\perp}\mathrm{tr}\left\{G^{+\perp}_{\rho}(i\partial_+)^{l-1}C^{5/2}_{j-1}\left(\frac{\mathcal{D}_+}{\partial_+}\right)G^{\perp+}_{\sigma}\right\}, \tag{23}$$

$$\widetilde{\mathcal{W}}^{\mu\nu}_{j,l} = i\tau^{\mu\nu,\rho\sigma}_{\perp}\mathrm{tr}\left\{\widetilde{G}^{+\perp}_{\rho}(i\partial_+)^{l-1}C^{5/2}_{j-1}\left(\frac{\mathcal{D}_+}{\partial_+}\right)G^{\perp+}_{\sigma}\right\}, \tag{24}$$

$\tau^{\mu\nu,\rho\sigma}_{\perp} = \frac{1}{2}\left(g^{\mu\rho}_{\perp}g^{\nu\sigma}_{\perp} + g^{\mu\sigma}_{\perp}g^{\nu\rho}_{\perp} - g^{\mu\nu}_{\perp}g^{\rho\sigma}_{\perp}\right)$. Note, that here we kept only terms containing at the end operator we are interested in.

We see one general property, which is inherent to this set of transformation: for bosonic operators with a certain parity ($\mathcal{S}^i$ or $\mathcal{P}^i$) the index j of final operator $\mathcal{W}^{\mu\nu}_{j,l}$ changes on two units after each step.

Using restricted supersymmetry transformation above, it is easy to find [22], that the renormalized supersymmetric Ward identity in the regularization scheme, preserving supersymmetry, has the following form ($\boldsymbol{\mathcal{S}}_{j,l}$ denotes vector of operators $S^1_{j,l}$, $S^2_{j,l}$ and $S^3_{j,l}$)

$$\langle[\boldsymbol{\mathcal{S}}_{jl}]\delta^Q\mathcal{X}\rangle = -\langle\delta^Q[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle - \langle i[\boldsymbol{\mathcal{S}}_{jl}](\delta^Q S)\mathcal{X}\rangle \qquad \text{and} \quad \langle\delta^Q[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle = \text{finite}\,, \tag{25}$$

where we used the fact, that renormalized action in supersymmetric regularization is invariant with respect to supersymmetry transformations $\langle i[\boldsymbol{\mathcal{S}}_{jl}](\delta^Q S)\mathcal{X}\rangle = 0$. As we already noted, beyond LO our operators (13), (15) and (17) mix under renormalization, so that the renormalized operators are defined as (square brackets correspond to renormalized quantities)

$$\left[\begin{array}{c}\mathcal{S}^1\\ \mathcal{S}^2\\ \mathcal{S}^3\end{array}\right]_{jl} = \sum_{k=0}^{j}\left(\begin{array}{ccc}{}^{11}Z_{\mathcal{S}} & {}^{12}Z_{\mathcal{S}} & {}^{13}Z_{\mathcal{S}}\\ {}^{21}Z_{\mathcal{S}} & {}^{22}Z_{\mathcal{S}} & {}^{23}Z_{\mathcal{S}}\\ {}^{31}Z_{\mathcal{S}} & {}^{32}Z_{\mathcal{S}} & {}^{33}Z_{\mathcal{S}}\end{array}\right)_{jk}\left(\begin{array}{ccc}Z^{-1}_{\phi} & 0 & 0\\ 0 & Z^{-1}_{\phi} & 0\\ 0 & 0 & Z^{-1}_{\phi}\end{array}\right)\left(\begin{array}{c}\mathcal{S}^1\\ \mathcal{S}^2\\ \mathcal{S}^3\end{array}\right)_{kl} \tag{26}$$

and as a consequence the renormalization group equation for these operators is given by

$$\frac{d}{d\ln\mu}\left[\begin{array}{c}\mathcal{S}^1\\ \mathcal{S}^2\\ \mathcal{S}^3\end{array}\right]_{jl} = \sum_{k=0}^{j}\left(\begin{array}{ccc}{}^{11}\gamma^{\mathcal{S}} & {}^{12}\gamma^{\mathcal{S}} & {}^{13}\gamma^{\mathcal{S}}\\ {}^{21}\gamma^{\mathcal{S}} & {}^{22}\gamma^{\mathcal{S}} & {}^{23}\gamma^{\mathcal{S}}\\ {}^{31}\gamma^{\mathcal{S}} & {}^{32}\gamma^{\mathcal{S}} & {}^{33}\gamma^{\mathcal{S}}\end{array}\right)_{jk}\left[\begin{array}{c}\mathcal{S}^1\\ \mathcal{S}^2\\ \mathcal{S}^3\end{array}\right]_{kl}. \tag{27}$$

Now, from supersymmetric Ward identity (25) we get ($\sigma_k = \frac{1}{2}(1-(-1)^k)$ and $Z_{jk} = 0$ for $k > j$)

$$\sum_{k=0}^{j}\sum_{k'=0}^{k}\left(\begin{array}{ccc}{}^{11}Z_{\mathcal{S}} & {}^{12}Z_{\mathcal{S}} & {}^{12}Z_{\mathcal{S}}\\ {}^{21}Z_{\mathcal{S}} & {}^{22}Z_{\mathcal{S}} & {}^{23}Z_{\mathcal{S}}\\ {}^{31}Z_{\mathcal{S}} & {}^{32}Z_{\mathcal{S}} & {}^{33}Z_{\mathcal{S}}\end{array}\right)_{jk}\sigma_k\left(\begin{array}{c}\{Z^{-1}_{\mathcal{W}}\}_{k-2,k'}\\ \{Z^{-1}_{\mathcal{W}}\}_{k,k'}\\ \{Z^{-1}_{\mathcal{W}}\}_{k+2,k'}\end{array}\right)[\mathcal{W}_{k'l}] = \text{finite}\,. \tag{28}$$

$1/\epsilon$ poles in (28) cancel, provided

$$\sum_{k=0}^{j}\left\{{}^{11}Z^{[1]}_{\mathcal{S}}\right\}_{jk}\sigma_k[\mathcal{W}_{k-2,l}] + \sum_{k=0}^{j}\left\{{}^{12}Z^{[1]}_{\mathcal{S}}\right\}_{jk}\sigma_k[\mathcal{W}_{k,l}] + \sum_{k=0}^{j}\left\{{}^{13}Z^{[1]}_{\mathcal{S}}\right\}_{jk}\sigma_k[\mathcal{W}_{k+2,l}] = \sigma_j\sum_{k=0}^{j}\left\{Z^{[1]}_{\mathcal{W}}\right\}_{j-2,k}[\mathcal{W}_{kl}],$$

$$\sum_{k=0}^{j}\left\{{}^{21}Z^{[1]}_{\mathcal{S}}\right\}_{jk}\sigma_k[\mathcal{W}_{k-2,l}] + \sum_{k=0}^{j}\left\{{}^{22}Z^{[1]}_{\mathcal{S}}\right\}_{jk}\sigma_k[\mathcal{W}_{k,l}] + \sum_{k=0}^{j}\left\{{}^{23}Z^{[1]}_{\mathcal{S}}\right\}_{jk}\sigma_k[\mathcal{W}_{k+2,l}] = \sigma_j\sum_{k=0}^{j}\left\{Z^{[1]}_{\mathcal{W}}\right\}_{j,k}[\mathcal{W}_{kl}],$$

$$\sum_{k=0}^{j}\left\{{}^{31}Z^{[1]}_{\mathcal{S}}\right\}_{jk}\sigma_k[\mathcal{W}_{k-2,l}] + \sum_{k=0}^{j}\left\{{}^{32}Z^{[1]}_{\mathcal{S}}\right\}_{jk}\sigma_k[\mathcal{W}_{k,l}] + \sum_{k=0}^{j}\left\{{}^{33}Z^{[1]}_{\mathcal{S}}\right\}_{jk}\sigma_k[\mathcal{W}_{k+2,l}] = \sigma_j\sum_{k=0}^{j}\left\{Z^{[1]}_{\mathcal{W}}\right\}_{j+2,k}[\mathcal{W}_{kl}].$$

Taking into account linear independence of operators $[\mathcal{W}_{kl}]$ we finally get the following relations on anomalous dimensions of conformal operators

$$
\begin{aligned}
{}^{11}\gamma^{\mathcal{S}}_{2n+5,2m+1} + {}^{12}\gamma^{\mathcal{S}}_{2n+5,2m-1} + {}^{13}\gamma^{\mathcal{S}}_{2n+5,2m-3} &= \gamma^{\mathcal{W}}_{2n+3,2m-1}, \qquad m \le n+2\,, &(29)\\
{}^{21}\gamma^{\mathcal{S}}_{2n+3,2m+1} + {}^{22}\gamma^{\mathcal{S}}_{2n+3,2m-1} + {}^{23}\gamma^{\mathcal{S}}_{2n+3,2m-3} &= \gamma^{\mathcal{W}}_{2n+3,2m-1}, \qquad m \le n+1\,, &(30)\\
{}^{31}\gamma^{\mathcal{S}}_{2n+1,2m+1} + {}^{32}\gamma^{\mathcal{S}}_{2n+1,2m-1} + {}^{33}\gamma^{\mathcal{S}}_{2n+1,2m-3} &= \gamma^{\mathcal{W}}_{2n+3,2m-1}, \qquad m \le n &(31)
\end{aligned}
$$

and

$$
\begin{aligned}
{}^{12}\gamma^{\mathcal{S}}_{2n+1,2n+1} + {}^{13}\gamma^{\mathcal{S}}_{2n+1,2n-1} &= 0\,, & {}^{13}\gamma^{\mathcal{S}}_{2n+1,2n+1} &= 0\,, &(32)\\
{}^{22}\gamma^{\mathcal{S}}_{2n+1,2n+1} + {}^{23}\gamma^{\mathcal{S}}_{2n+1,2n-1} &= \gamma^{\mathcal{W}}_{2n+1,2n+1}\,, & {}^{23}\gamma^{\mathcal{S}}_{2n+1,2n+1} &= 0\,, &(33)\\
{}^{32}\gamma^{\mathcal{S}}_{2n+1,2n+1} + {}^{33}\gamma^{\mathcal{S}}_{2n+1,2n-1} &= \gamma^{\mathcal{W}}_{2n+3,2n+1}\,, & {}^{33}\gamma^{\mathcal{S}}_{2n+1,2n+1} &= \gamma^{\mathcal{W}}_{2n+3,2n+3}\,. &(34)
\end{aligned}
$$

Now, let us turn to polarized case. All the steps one need to perform here are in one to one correspondence with unpolarized case. First we write down the supersymmetric Ward identity ($\boldsymbol{\mathcal{P}}_{j,l}$ denotes vector of operators $P^1_{j,l}$ (14) and $P^2_{j,l}$ (16))

$$
\langle[\boldsymbol{\mathcal{P}}_{jl}]\delta^Q\mathcal{X}\rangle = -\langle\delta^Q[\boldsymbol{\mathcal{P}}_{jl}]\mathcal{X}\rangle - \langle i[\boldsymbol{\mathcal{P}}_{jl}](\delta^Q P)\mathcal{X}\rangle \qquad \text{and} \quad \langle\delta^Q[\boldsymbol{\mathcal{P}}_{jl}]\mathcal{X}\rangle = \text{finite}\,. \tag{35}
$$

The operators (14) and (16) mix under renormalization and thus we define renormalized operators as (square brackets correspond to renormalized quantities)

$$
\left[\begin{array}{c}\mathcal{P}^1\\ \mathcal{P}^2\end{array}\right]_{jl} = \sum_{k=0}^{j}\left(\begin{array}{cc}{}^{11}Z_{\mathcal{P}} & {}^{12}Z_{\mathcal{P}}\\ {}^{21}Z_{\mathcal{P}} & {}^{22}Z_{\mathcal{P}}\end{array}\right)_{jk}\left(\begin{array}{cc}Z_\phi^{-1} & 0\\ 0 & Z_\phi^{-1}\end{array}\right)\left(\begin{array}{c}\mathcal{P}^1\\ \mathcal{P}^2\end{array}\right)_{kl}. \tag{36}
$$

The renormalization group equation for these operators is given by

$$
\frac{d}{d\ln\mu}\left[\begin{array}{c}\mathcal{P}^1\\ \mathcal{P}^2\end{array}\right]_{jl} = \sum_{k=0}^{j}\left(\begin{array}{cc}{}^{11}\gamma^{\mathcal{P}} & {}^{12}\gamma^{\mathcal{P}}\\ {}^{21}\gamma^{\mathcal{P}} & {}^{22}\gamma^{\mathcal{P}}\end{array}\right)_{jk}\left[\begin{array}{c}\mathcal{P}^1\\ \mathcal{P}^2\end{array}\right]_{kl}. \tag{37}
$$

From supersymmetric Ward identity (35) we get ($\bar\sigma_k = \frac{1}{2}(1+(-1)^k)$ and $Z_{jk} = 0$ for $k > j$)

$$
\sum_{k=0}^{j}\sum_{k'=0}^{k}\left(\begin{array}{cc}{}^{11}Z_{\mathcal{P}} & {}^{12}Z_{\mathcal{P}}\\ {}^{21}Z_{\mathcal{P}} & {}^{22}Z_{\mathcal{P}}\end{array}\right)_{jk}\bar\sigma_k\left(\begin{array}{c}\{Z^{-1}_{\widetilde{\mathcal{W}}}\}_{k-1,k'}\\ \{Z^{-1}_{\widetilde{\mathcal{W}}}\}_{k+1,k'}\end{array}\right)[\widetilde{\mathcal{W}}_{k'l}] = \text{finite}\,. \tag{38}
$$

$1/\epsilon$ poles in (38) cancel, provided

$$
\begin{aligned}
\sum_{k=0}^{j}\left\{{}^{11}Z^{[1]}_{\mathcal{P}}\right\}_{jk}\bar\sigma_k[\widetilde{\mathcal{W}}_{k-1,l}] + \sum_{k=0}^{j}\left\{{}^{12}Z^{[1]}_{\mathcal{P}}\right\}_{jk}\bar\sigma_k[\widetilde{\mathcal{W}}_{k+1,l}] &= \bar\sigma_j\sum_{k=0}^{j}\left\{Z^{[1]}_{\widetilde{\mathcal{W}}}\right\}_{j-1,k}[\widetilde{\mathcal{W}}_{kl}],\\
\sum_{k=0}^{j}\left\{{}^{21}Z^{[1]}_{\mathcal{P}}\right\}_{jk}\bar\sigma_k[\widetilde{\mathcal{W}}_{k-1,l}] + \sum_{k=0}^{j}\left\{{}^{22}Z^{[1]}_{\mathcal{P}}\right\}_{jk}\bar\sigma_k[\widetilde{\mathcal{W}}_{k+1,l}] &= \bar\sigma_j\sum_{k=0}^{j}\left\{Z^{[1]}_{\widetilde{\mathcal{W}}}\right\}_{j+1,k}[\widetilde{\mathcal{W}}_{kl}].
\end{aligned}
$$

Taking into account linear independence of operators $[\widetilde{\mathcal{W}}_{kl}]$ we finally get the following relations

$$
\begin{aligned}
{}^{11}\gamma^{\mathcal{P}}_{2n+2,2m} + {}^{12}\gamma^{\mathcal{P}}_{2n+2,2m-2} &= \gamma^{\widetilde{\mathcal{W}}}_{2n,2m-2}\,, \qquad m \le n+1\,, &(39)\\
{}^{21}\gamma^{\mathcal{P}}_{2n,2m} + {}^{22}\gamma^{\mathcal{P}}_{2n,2m-2} &= \gamma^{\widetilde{\mathcal{W}}}_{2n,2m-2}\,, \qquad m \le n &(40)
\end{aligned}
$$

and

$$ {}^{12}\gamma^{\mathcal{P}}_{2n,2n} = 0\,, \qquad\qquad {}^{22}\gamma^{\mathcal{P}}_{2n,2n} = \gamma^{\widetilde{\mathcal{W}}}_{2n,2n}\,. \tag{41} $$

Note, that both operators $W_{j,k}$ and $\widetilde{W}_{j,k}$ are components of the same operator supermultiplet and thus have the same single anomalous dimension (with shifted argument).

To obtain from these results the corresponding relations for anomalous dimensions of conformal operators considered earlier, one just need to use the following transition formulae among anomalous dimensions of conformal operators (13)-(17) and anomalous dimensions of operators (7)-(11):

$$ \begin{pmatrix} {}^{11}\gamma^{\mathcal{P}} & {}^{12}\gamma^{\mathcal{P}} \\ {}^{21}\gamma^{\mathcal{P}} & {}^{22}\gamma^{\mathcal{P}} \end{pmatrix} = \begin{pmatrix} 6 & \frac{j}{2} \\ 6 & -\frac{j+3}{2} \end{pmatrix} \begin{pmatrix} {}^{gg}\tilde{\gamma} & {}^{g\lambda}\tilde{\gamma} \\ {}^{\lambda g}\tilde{\gamma} & {}^{\lambda\lambda}\tilde{\gamma} \end{pmatrix} \begin{pmatrix} 6 & \frac{k}{2} \\ 6 & -\frac{k+3}{2} \end{pmatrix}^{-1}, \tag{42} $$

$$ \begin{pmatrix} {}^{11}\gamma^{\mathcal{S}} & {}^{12}\gamma^{\mathcal{S}} & {}^{13}\gamma^{\mathcal{S}} \\ {}^{21}\gamma^{\mathcal{S}} & {}^{22}\gamma^{\mathcal{S}} & {}^{23}\gamma^{\mathcal{S}} \\ {}^{31}\gamma^{\mathcal{S}} & {}^{32}\gamma^{\mathcal{S}} & {}^{33}\gamma^{\mathcal{S}} \end{pmatrix} = \begin{pmatrix} 6 & \frac{j}{2} & \frac{j(j+1)}{4} \\ 6 & -\frac{1}{4} & -\frac{(j+1)(j+2)}{12} \\ 6 & -\frac{j+3}{2} & \frac{(j+2)(j+3)}{4} \end{pmatrix} \begin{pmatrix} {}^{gg}\gamma & {}^{g\lambda}\gamma & {}^{g\phi}\gamma \\ {}^{\lambda g}\gamma & {}^{\lambda\lambda}\gamma & {}^{\lambda\phi}\gamma \\ {}^{\phi g}\gamma & {}^{\phi\lambda}\gamma & {}^{\phi\phi}\gamma \end{pmatrix} \begin{pmatrix} 6 & \frac{k}{2} & \frac{k(k+1)}{4} \\ 6 & -\frac{1}{4} & -\frac{(k+1)(k+2)}{12} \\ 6 & -\frac{k+3}{2} & \frac{(k+2)(k+3)}{4} \end{pmatrix}^{-1}. \tag{43} $$

These relations allow us to operate both with anomalous dimensions of conformal operators (7)-(11) or, equivalently, with anomalous dimensions of multiplicatively renormalized operators (13)-(17).

3 Universal non-forward AD in $N = 4$ SUSY YM.

Now, let us proceed with the determination of universal non-forward anomalous dimension $\gamma^{uni}_{j,k} = \gamma^{\mathcal{W}}_{j,k}$. The simplest way to do it is to use the formalism of broken conformal Ward identities (CWI) Ref. [18, 19]. The basic idea of this method lies in the relation between scale and special conformal anomalies of conformal operators. It was found that non-diagonal part $\gamma^{\rm ND}$ of the complete anomalous dimensions matrix

$$ \gamma_{j,k} = \gamma^{\rm D}_j \delta_{j,k} + \gamma^{\rm ND}_{j,k}\,, \qquad\qquad \gamma = \frac{\alpha_s N_c}{2\pi}\gamma^{(0)} + \left(\frac{\alpha_s N_c}{2\pi}\right)^2 \gamma^{(1)} + ... \tag{44} $$

arises entirely due to the violation of special conformal symmetry. Moreover, within this framework we have a relation between scale anomalous dimension matrix in n-th order of perturbation theory and matrices of scale and special conformal anomalies in $(n-1)$-order of perturbation theory. As a consequence the calculation of leading nondiagonal part of nonforward anomalous dimensions matrix, which is nonzero starting from two-loop order, could be reduced to calculation of more simple one-loop diagrams.

Up to the moment there are results for nondiagonal parts of nonforward anomalous dimension matrices of conformal operators calculated in QED [20], QCD [21], $N = 1$ supersymmetric Yang-Mills theory [22] and in supersymmetric Wess-Zumino model [36], both for polarized and unpolarized cases. The results in $N = 1$ SYM could be easily derived from already known QCD result via simple identification of colour Casimir operators: $C_A = C_F = 2N_f T_F = N_c$. In $N = 4$ SYM the situation is slightly more complicated. Besides correspondingly adjusted map of colour Casimir operators:

$C_A = C_F = \frac{1}{2}N_f T_F = N_c$, where we accounted for four Majorana fermions compared to one in $N = 1$ SYM, we need to consider scalar fields ϕ, which are absent in $N = 1$ SYM. The presence of scalar fields results in introduction of one additional conformal operator (17) and increases the size of anomalous dimension matrix in unpolarized case. So, in general, in unpolarized case we need to compute 5 new conformal anomalies. However, as we already noted, to find leading nondiagonal part of nonforward anomalous dimensions matrix all we need to know is one-loop matrix of special conformal anomalies (matrix of scale anomalies is diagonal at one-loop order and is known for a long time). A simple diagram analysis shows, that the only nontrivial contribution to special conformal anomaly matrix, not related to one-loop scale anomaly matrix and not already computed in QCD, resides in ${}^{\phi\phi}\gamma$ and ${}^{g\phi}\gamma$.

On the contrary, in polarized case we have only two conformal operators: (8) and (10), which are similar to polarized operators in $N = 1$ SYM and QCD. In this case we do not need to perform any new calculations and we can use polarized case to extract universal non-diagonal part of non-forward anomalous dimension matrix from already known results for non-forward polarized anomalous dimension matrix in QCD from Refs. [19, 20, 21] and results for leading order forward polarized anomalous dimension matrix in $N = 4$ SYM from Refs. [10, 6]. As a result we obtain the following final expressions for leading non-diagonal parts of non-forward polarized anomalous dimension matrix in $N = 4$ SYM ($j > k$):

$$
\begin{aligned}
{}^{\lambda\lambda}\widetilde{\gamma}^{\mathrm{ND}(1)}_{jk} &= -\left({}^{\lambda\lambda}\widetilde{\gamma}^{(0)}_j - {}^{\lambda\lambda}\widetilde{\gamma}^{(0)}_k\right)\left(d_{jk}{}^{\lambda\lambda}\widetilde{\gamma}^{(0)}_k - {}^{\lambda\lambda}g_{jk}\right) \\
&\quad - \left({}^{\lambda g}\widetilde{\gamma}^{(0)}_j - {}^{\lambda g}\widetilde{\gamma}^{(0)}_k\right) d_{jk}{}^{g\lambda}\widetilde{\gamma}^{(0)}_k + {}^{\lambda g}\widetilde{\gamma}^{(0)}_j{}^{g\lambda}g_{jk}\,, && (45)\\
{}^{\lambda g}\widetilde{\gamma}^{\mathrm{ND}(1)}_{jk} &= -\left({}^{\lambda g}\widetilde{\gamma}^{(0)}_j - {}^{\lambda g}\widetilde{\gamma}^{(0)}_k\right) d_{jk}{}^{gg}\widetilde{\gamma}^{(0)}_k - \left({}^{\lambda\lambda}\widetilde{\gamma}^{(0)}_j - {}^{\lambda\lambda}\widetilde{\gamma}^{(0)}_k\right) d_{jk}{}^{\lambda g}\widetilde{\gamma}^{(0)}_k \\
&\quad + {}^{\lambda g}\widetilde{\gamma}^{(0)}_j{}^{gg}g_{jk} - {}^{\lambda\lambda}g_{jk}{}^{\lambda g}\widetilde{\gamma}^{(0)}_k\,, && (46)\\
{}^{g\lambda}\widetilde{\gamma}^{\mathrm{ND}(1)}_{jk} &= -\left({}^{g\lambda}\widetilde{\gamma}^{(0)}_j - {}^{g\lambda}\widetilde{\gamma}^{(0)}_k\right) d_{jk}{}^{\lambda\lambda}\widetilde{\gamma}^{(0)}_k - \left({}^{gg}\widetilde{\gamma}^{(0)}_j - {}^{gg}\widetilde{\gamma}^{(0)}_k\right) d_{jk}{}^{g\lambda}\widetilde{\gamma}^{(0)}_k \\
&\quad + {}^{g\lambda}\widetilde{\gamma}^{(0)}_j{}^{\lambda\lambda}g_{jk} - {}^{gg}g_{jk}{}^{g\lambda}\widetilde{\gamma}^{(0)}_k + \left({}^{gg}\widetilde{\gamma}^{(0)}_j - {}^{\lambda\lambda}\widetilde{\gamma}^{(0)}_k\right){}^{g\lambda}g_{jk}\,, && (47)\\
{}^{gg}\widetilde{\gamma}^{\mathrm{ND}(1)}_{jk} &= -\left({}^{gg}\widetilde{\gamma}^{(0)}_j - {}^{gg}\widetilde{\gamma}^{(0)}_k\right)\left(d_{jk}{}^{gg}\widetilde{\gamma}^{(0)}_k - {}^{gg}g_{jk}\right) \\
&\quad - \left({}^{g\lambda}\widetilde{\gamma}^{(0)}_j - {}^{g\lambda}\widetilde{\gamma}^{(0)}_k\right) d_{jk}{}^{\lambda g}\widetilde{\gamma}^{(0)}_k - {}^{g\lambda}g_{jk}{}^{\lambda g}\widetilde{\gamma}^{(0)}_k\,. && (48)
\end{aligned}
$$

where $d_{jk} = b(j,k)/a(j,k)$, $g_{jk} = w_{jk}/a(j,k)$ and

$$
\begin{aligned}
a(j,k) &= 2(j-k)(j+k+3), \\
b(j,k) &= -[1+(-1)^{j-k}](2k+3), \quad \text{if } j > k \text{ and even}\,;\ 0 \text{ otherwise}\,.
\end{aligned}
$$

The quantities $w_{j,k}$ were computed in Refs. [19, 20, 21] and are given by

$$
\begin{aligned}
{}^{gg}w_{j,k} &= -2[1+(-1)^{j-k}]\theta_{j-2,k}(3+2k) \\
&\quad \times \left(2A_{jk} + (A_{jk} - S_1(j+1))\left[\frac{(j)_4}{(k)_4} - 1\right] + 2(j-k)(j+k+3)\frac{1}{(k)_4}\right), \qquad (49) \\
{}^{\lambda\lambda}w_{jk} &= -2[1+(-1)^{j-k}]\theta_{j-2,k}(3+2k) \\
&\quad \times \left(2A_{jk} + (A_{jk} - S_1(j+1))\frac{(j-k)(j+k+3)}{(k+1)(k+2)}\right), \qquad (50) \\
{}^{g\lambda}w_{jk} &= -2[1+(-1)^{j-k}]\theta_{j-2,k}(3+2k)\frac{1}{6}\frac{(j-k)(j+k+3)}{(k+1)(k+2)}, \qquad (51) \\
{}^{\lambda g}w_{jk} &= 0, \qquad (52)
\end{aligned}
$$

where $\theta_{j,k}$ is equal to 1, if $j > k$ and 0 otherwise and

$$
A_{jk} = S_1\left(\frac{j+k+2}{2}\right) - S_1\left(\frac{j-k-2}{2}\right) + 2S_1(j-k-1) - S_1(j+1). \qquad (53)
$$

Here $S_k(n)$ denotes harmonic sum

$$
S_i(j) = \sum_{m=1}^{j} \frac{1}{m^i}, \qquad S_{-i}(j) = (-1)^j \sum_{m=1}^{j} \frac{(-1)^m}{m^i}, \qquad S_{-2,1}(j) = (-1)^j \sum_{m=1}^{j} \frac{(-1)^m}{m^2} S_1(m) \qquad (54)
$$

and $(a)_k = \frac{\Gamma(a+k)}{\Gamma(a)}$ stands for Pochhammer symbol. The leading order forward anomalous dimensions of conformal operators, entering formula above are given by [10, 6]

$$
\begin{aligned}
{}^{\lambda\lambda}\widetilde{\gamma}_j^{(0)} &= 4\left(\frac{1}{(j+1)(j+2)} + S_1(j+1)\right), \qquad (55) \\
{}^{\lambda g}\widetilde{\gamma}_j^{(0)} &= -\frac{48}{(j+1)(j+2)}, \qquad (56) \\
{}^{g\lambda}\widetilde{\gamma}_j^{(0)} &= -\frac{j(j+3)}{3(j+1)(j+2)}, \qquad (57) \\
{}^{gg}\widetilde{\gamma}_j^{(0)} &= 4S_1(j+1) + \frac{8}{(j+1)(j+2)}. \qquad (58)
\end{aligned}
$$

Collecting all contributions together and using the relations (42) and (39) (or (40)) we find the following expression for universal non-forward anomalous dimension in $N = 4$ SYM

$$
\begin{aligned}
{}^{uni}\gamma_{j,k}^{\mathrm{ND}(1)} &= \gamma_{j-2,k-2}^{\widetilde{\mathcal{W}}} = 4\frac{\big(1+(-1)^{j-k}\big)(2k+1)\big(S_1(j)-S_1(k)\big)}{(k-1)_4(j-k)(j+k+1)} \times \\
&\times \left\{\left[S_1\!\left(\frac{j-k}{2}\right) - 2S_1(j-k) - S_1\!\left(\frac{j+k}{2}\right)\right]\Big((j-1)_4 + (k-1)_4\Big)\right. \\
&\quad \left. + 2(j-1)_4 S_1(j) + 2(k-1)_4 S_1(k)\right\}. \qquad (59)
\end{aligned}
$$

Now, combining Eqs. (39) and (40) together we get

$$ {}^{11}\gamma^{\mathcal{P}}_{2n+2,2m} + {}^{12}\gamma^{\mathcal{P}}_{2n+2,2m-2} - {}^{21}\gamma^{\mathcal{P}}_{2n,2m} - {}^{22}\gamma^{\mathcal{P}}_{2n,2m-2} = 0\,. \tag{60} $$

Taking $m = n-1$ it is easy to check using Eq. (42), that our nonforward anomalous dimensions satisfy this relation. Moreover we can check relation between the NLO non-forward anomalous dimension and NLO forward anomalous dimension from Ref. [7] [3]. Namely from Eq. (60) we have

$$ {}^{11}\gamma^{\mathcal{P}}_{2n+2,2n} + {}^{12}\gamma^{\mathcal{P}}_{2n+2,2n-2} - {}^{21}\gamma^{\mathcal{P}}_{2n,2n} - {}^{22}\gamma^{\mathcal{P}}_{2n,2n-2} = 0\,, \qquad \text{for} \quad m = n\,, \tag{61} $$

$$ {}^{11}\gamma^{\mathcal{P}}_{2n,2n} + {}^{12}\gamma^{\mathcal{P}}_{2n,2n-2} - {}^{22}\gamma^{\mathcal{P}}_{2n-2,2n-2} = 0\,, \qquad \text{for} \quad m = n+1\,. \tag{62} $$

Indeed, taking explicit expressions for NLO forward anomalous dimensions from Ref. [7] (to adapt these results to normalization used in present paper one needs to multiply the results of Ref. [7] by $(-\frac{1}{2})$, shift the value of momentum j by unity $j = j+1$ and then make the following substitutions: ${}^{\lambda g}\tilde{\gamma}_j = {}^{\lambda g}\tilde{\gamma}_j\,\frac{6}{j}$ and ${}^{g\lambda}\tilde{\gamma}_j = {}^{g\lambda}\tilde{\gamma}_j\,\frac{j}{6}$)

$$ {}^{11}\gamma^{\mathcal{P}}_{j,j} \;=\; {}^{22}\gamma^{\mathcal{P}}_{j-2,j-2} \;=\; -4\big(S_3(j) + S_{-3}(j)\big) + 8S_1(j)\big(S_2(j) + S_{-2}(j)\big) - 8S_{-2,1}(j)\,, \tag{63} $$

$$ {}^{21}\gamma^{\mathcal{P}}_{j,j} \;=\; \frac{24 S_1(j)}{j(j+1)(j+2)} - \frac{8(2j+3)}{(j+1)^2(j+2)^2}\,. \tag{64} $$

where $S_i(j)$ stands for harmonic sum defined in Eq. (54), one can easily verify, that above relations hold true. Note, that Eq. (61) contains a contribution from forward non-diagonal anomalous dimension ${}^{21}\gamma_{2n,2n}$. The appearance of nondiagonal part of forward anomalous dimension matrix (42) is related to the breaking of superconformal symmetry, what is explicitly demonstrated by Eq. (61).

Finally, having in mind possible future tests of AdS/CFT correspondence, we would like to consider one particular limit of our result Eq. (59). As this result is meaningful only for $j > k$ we parameterize the limit of large j and k as follows:

$$ j = n\,m\,, \qquad\qquad k = m\,. \tag{65} $$

Now, using $S_1(j) = \Psi(j+1) - \Psi(1) \approx \ln(j)$ for large j, it is easy to find the following asymptotic behaviour of Eq. (59) (for even $j-k$)

$$ {}^{uni}\gamma^{\mathrm{ND}(1)}_{n\,m,m} = \frac{16}{m}\frac{\ln(n)}{n^2-1}\left[n^4 \ln\left(\frac{n^2}{n^2-1}\right) + \ln\left(\frac{1}{n^2-1}\right)\right]. \tag{66} $$

4 Conclusion

In the present paper we found a closed analytical expression for NLO universal non-forward anomalous dimension of Wilson twist-2 operators in $N=4$ supersymmetric Yang-Mills theory. To derive

[3] In Ref. [7] the results of calculation were written in DRED scheme, however the coupling constant was kept in $\overline{\mathrm{MS}}$ scheme. To obtain a result fully expressed in DRED quantities, one needs to perform the following coupling constant redefinition for coupling constant $\alpha_{\mathrm{DRED}} = \alpha_{\overline{\mathrm{MS}}} + N_c/(12\pi)\,(\alpha_{\overline{\mathrm{MS}}})^2$. It means, that we need to add to NLO anomalous dimension from Ref. [7] the corresponding LO expression from Refs. [10, 6], that is in present paper we are using $\gamma^{(1)}_j = \left[\gamma^{(1)}_j\right]_{\mathrm{Ref.}[7]} - 1/3\,\gamma^{(0)}_j$

this result it was sufficient to know constrains on anomalous dimensions of conformal operators, following from supersymmetric Ward identities, together with already known results for anomalous dimensions of Wilson twist-2 operators in QCD. Moreover, supersymmetric Ward identities allowed us to find the relation of non-forward anomalous dimension determined here with already known forward anomalous dimension of Wilson twist-2 operators in $N = 4$ SYM theory. This relation may serve as an additional check of correctness of the results obtained both for forward and non-forward anomalous dimensions of conformal operators in this model up to next-to-leading order in perturbation theory. Finally, we considered the limit of our result for non-forward anomalous dimension, when momenta j and k turn to infinity, while their ratio is being fixed. In this limit non-forward anomalous dimension scales as $1/j$, where j is Lorentz spin.

Acknowledgments

We are grateful to L.N. Lipatov for numerous discussions on the subject of this paper. The work of A.O. was supported by the National Science Foundation under grant PHY-0244853 and by the US Department of Energy under grant DE-FG02-96ER41005. The work of V.V. is supported by grants INTAS 00-366 and RSGSS-1124.2003.2.

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